OECD OCDE Organisation de Coopération et de Développement Economiques Organisation for Economic Co-operation and Development	A World Free of Poverty THE WORLD BANK GROUP

Using Price and Quantity Indicators to Explore Data Structure

Mr. Jim Cuthbert, Edinburgh - UK

Agenda item n° 13

JOINT WORLD BANK – OECD SEMINAR ON PURCHASING POWER PARITIES

Recent Advances in Methods and Applications

WASHINGTON D.C.

30 January – 2 February 2001

Summary

This paper shows how it is possible to define indicators of price and quantity structure, which can be used to explore the price and quantity data sets used in PPP work. The indicators have a number of potential applications, including as an aid in data vetting, and can be used to reveal structural features of the data which may be of intrinsic interest in their own right. Moreover, these structural features can be related to the comparative performance of different aggregation techniques. The definition and application of these indicators is illustrated with reference to the 1993 OECD data.

1. Introduction

1.1 One of the puzzling aspects of PPP research is that much attention appears to be paid to studying the properties of different aggregation methods, and to devising new methods of aggregation, while relatively little attention appears to be paid to understanding the key structural features of our main price and quantity data sets. This imbalance is potentially damaging, since structural features of the data are likely to play a fundamental role in determining the relative properties of different aggregation methods. In other words, we probably cannot understand about aggregation methods until we fully understand the data.

1.2 This paper shows how it is possible to define indicators of price and quantity structure which can be used in data vetting, and to explore structural features of the basic data: and illustrates how such structural features can be related to the comparative performance of different aggregation techniques. The data set considered is the set of price and quantity data used from basic heading up in the 1993 OECD aggregation exercise: (I am grateful to the OECD for making this data set available to me.) Fuller information, including more technical details, is contained in the paper by Cuthbert, [2000], published last November in the Journal of the Royal Statistical Society.

1.3 The structure of this paper is as follows:

Section 2 is a brief preamble, which introduces concepts related to Generalised Geary Khamis (GGK) indices, required for the subsequent definition of the indicators studied in this paper.

Section 3 introduces and defines the price and quantity indicators to be used.

Section 4 then illustrates the application of these indicators to the 1993 OECD data set. The use of the indicators to identify extreme values is illustrated. The main application, however, is to reveal key structural features of the dataset.

Section 5 then relates these observed structural features to the relative performance of a number of different aggregation techniques, including the GEKS, the Geary Khamis, the Iklé, the equal weighted Van Ijzeren, and a group of Generalised Geary Khamis indices.

Section 6 looks at the robustness of the techniques developed here, and concludes that they are robust.

1.4 Finally, on a logistical note. This paper is illustrated by a number of charts: these have not been included with this electronic version of the paper, but will be circulated at the Washington meeting.

2. Introducing GGK indices

2.1 This paper is not primarily concerned with the theory of Generalised Geary Khamis, (GGK), indices. However, for reasons which will be discussed later, the price and quantity indicators which are our primary concern are most sensibly defined relative to a given GGK index. It is therefore necessary to start with a brief introduction to GGK indices. For full details, see Cuthbert, [1999] and [2000].

2.2 Let p_{ij} and q_{ij} denote, respectively, the price and quantity of commodity i in the j'th country, (where there are J countries, and I items (or commodities) included in the comparison.)

2.3 As is well known, the standard Geary Khamis, (GK), method, defined by Geary, [1958], defines international prices π_i and expenditure deflators e_j in terms of solutions to the following equations:

$$\pi_i = \frac{\sum_j e_j p_{ij} q_{ij}}{\sum_j q_{ij}}, \quad \text{for all } i \qquad (1)$$

$$e_j = \frac{\sum_i \pi_i q_{ij}}{\sum_i p_{ij} q_{ij}}, \quad \text{for all } j. \qquad (2)$$

2.4 Equation (2) above means that the GK index, by definition, satisfies the property that

$$e_j \sum_i p_{ij} q_{ij} = \sum_i \pi_i q_{ij}, \quad \text{for all } j:$$

in other words, real volumes, as computed directly from GK international prices, are the same as deflated expenditures. This important property is known as strong additivity, as defined in Cuthbert, [1999]. Cuthbert categorised all strongly additive indices within a broad general class of aggregation methods - and proved that, under reasonable conditions, the set of strongly additive indices is identical to what he denoted as the set of Generalised Geary Khamis indices, (GGK indices), defined as follows.

Definition: Let **b** be a vector of positive quantities β_j. Then the GGK aggregation method corresponding to **b** defines the quantities π_i and e_j as the solutions to the equations

$$\pi_i = \frac{\sum_j e_j p_{ij} \beta_j q_{ij}}{\sum_j \beta_j q_{ij}}, \text{ for all } i \qquad (3)$$

$$e_j = \frac{\sum_i \pi_i q_{ij}}{\sum_i p_{ij} q_{ij}}, \qquad \text{for all } j. \qquad (4)$$

2.5 Within this broad class of GGK indices, a particular sub-group plays an important role. For α in the range $0 \le \alpha \le 1$, choose β_j so that, in addition to equations (3) and (4), β_j satisfies

$$\beta_j = \frac{1}{\left[\sum_i \pi_i q_{ij}\right]^\alpha} \qquad (5)$$

Then the GGK index satisfying equations (3), (4) and (5) is denoted here as the $C(\alpha)$ index. It can be proved that the $C(\alpha)$ index always exists, for positive p_{ij} and q_{ij}.

When $\alpha = 0$, then the C(0) index is simply the standard GK index.

When $\alpha = 1$, then the C(1) index is identical to another standard index, namely, that introduced by Iklé , [1972]: this index attaches equal overall weight to each country in calculating international prices.

For $0 < \alpha < 1$, the $C(\alpha)$ represents a range of indices, lying between the GK and the Iklé. Equation (5) means that, as α increases, progressively greater weight is attached to small countries in the calculation of international prices.

2.6 The important point about GGK indices, (indeed, the feature which characterises GGK indices), is that there is no constant of proportionality in equation (4). Contrast this, for example, with an index like the equal weighted Van Ijzeren index: (see Van Ijzeren, [1987]). This is defined in terms of solutions to the following equations in π_i, e_j and λ, namely

$$\pi_i = \frac{1}{J} \cdot \sum_j e_j p_{ij}, \quad \text{for all } i \qquad (6)$$

$$e_j = \lambda \frac{\sum_i \pi_i q_{ij}}{\sum_i p_{ij} q_{ij}}, \qquad \text{for all } j. \qquad (7)$$

The Van Ijzeren index is not a GGK index, and so is not strongly additive: in other words, the term λ in equation (7) will not in general be equal to 1.

2.7 It is this feature of GGK indices, strong additivity, which, as we shall see, makes them particularly appropriate as a basis for defining indicators of price and quantity structure.

3. Indicators of Price and Quantity Structure

3.1 A basic problem in analysing price or quantity data arises because of the arbitrary units in which the quantities of the different items are expressed. The quantities of each item are, of course, expressed in terms of particular units appropriate for that item: these might either be physical units, (e.g., tons of steel), or implicit units, (e.g., the quantity of the specific commodity which could be bought by one unit of a base countries' currency in the base country). Of course, one of the requirements of any sensible aggregation method is that the results should be independent of the particular set of units in which the basic quantities are expressed. All standard aggregation methods satisfy this property- so, as regards comparing the results of different aggregation methods, there is no need to worry about the original choice of quantity unit. However, for other purposes, the choice of unit is important. For example, if it were desired to study the similarity between the price structures in pairs of countries by looking at the correlation between prices in these countries, then the results will be highly dependent on the choice of quantity units for the different items: so dependent, in fact, that simply calculating correlations between the raw price vectors is essentially meaningless.

3.2 To get round this problem, it would be desirable to be able to define indicators of price and quantity structure, which were independent of the arbitrary choice of unit in which the different quantities are expressed. Ideally, such indicators would also be independent of arbitrary choice of base country.

3.3 Let π_i and e_j be the international prices and price deflators arising from a given GGK aggregation method. Then this paper employs indicators of price and quantity structure, (denoted IP and IQ respectively), defined as

$$IP_{ij} = \frac{e_j p_{ij}}{\pi_i} \qquad (8)$$

$$IQ_{ij} = \frac{q_{ij} \cdot \sum_{i,j} \pi_i q_{ij}}{\sum_j q_{ij} \cdot \sum_i \pi_i q_{ij}}, \qquad (9)$$

The rationale for the definition of IP given in equation (8) is as follows. The simplest model of price structure is a multiplicative model, in which p_{ij} is determined as the product of a country effect and an item effect. If this model held, then p_{ij} would simply be equal to $e_j^{-1}.\pi_i$. It follows that IP_{ij}, as defined in (8), represents the ratio of the actual value of p_{ij} to what would be expected under the simple multiplicative model. A value of IP_{ij} greater than 1 suggests that the observed value of p_{ij} is relatively high, compared to what would be expected from the simple multiplicative model- and vice versa.

3.4 The rationale for the definition of IQ in equation (9) is similar. The simplest model for the structure of real volumes, $\pi_i q_{ij}$, would be a multiplicative model, in which $\pi_i q_{ij}$ would be equal to the country j share of world volume, (that is, $\frac{\sum_i \pi_i q_{ij}}{\sum_{i,j} \pi_i q_{ij}}$), times the item i share of world volume, (that is, $\frac{\sum_j \pi_i q_{ij}}{\sum_{i,j} \pi_i q_{ij}}$), times the world volume, (that is, $\sum_{i,j} \pi_i q_{ij}$). The indicator IQ_{ij} then represents the ratio of the actual volume $\pi_i q_{ij}$ to the expected volume under the simple multiplicative model. Again, a value of IQ greater than 1 suggests that q_{ij} is relatively large, and vice versa.

3.5 It can be verified that the indicators IP and IQ are independent of the choice of quantity units in which the data are originally expressed. In addition, IP and IQ are also independent of the choice of numeraire country in the aggregation. The indicators are, however, defined in terms of the π_i and e_j resulting from a specific choice of base GGK. It might be objected, therefore, that use of IP and IQ simply substitutes one form of dependency, (on the base aggregation method chosen), for another, (i.e., the choice of quantity units). As will be illustrated in Section 6, however, the main structural features of the data revealed by IP and IQ are robust to variations in the choice of base GGK.

3.6 Finally, the above indicators of price and quantity structure, IP and IQ, have been defined relative to some GGK index as base. Why is it particularly appropriate to use a GGK index for this purpose, rather than some other choice of base index? The primary reason relates to strong additivity:

As we have noted, a value of IP_{ij} greater than 1 suggests that the observed value of p_{ij} is relatively high - and vice versa, (and similarly for IQ as regards q_{ij}). It is therefore important that the array of IP values should indeed be well centred around 1. When IP is defined in terms of a GGK base, then this is the case, in the sense that a particular weighted average of each row and column of the IP array is equal to 1. This follows from defining equations (3) and (4) above for a GGK, which imply respectively that

$$\sum_j IP_{ij}\phi_{ij} = 1 \text{ , for all i, where } \phi_{ij} = \frac{\beta_j q_{ij}}{\sum_j \beta_j q_{ij}}, \qquad (10)$$

and

$$\sum_i IP_{ij}\theta_{ij} = 1 \text{ , for all j, where } \theta_{ij} = \frac{\pi_i q_{ij}}{\sum_i \pi_i q_{ij}}, \qquad (11)$$

Contrast this with what would happen if IP was defined relative to a base index which was not strongly additive. For example, if the equal weighted Van Ijzeren were used, then from equation (7) it follows that equation (11) would be replaced by the following,

$$\sum_i IP_{ij}\theta_{ij} = \lambda \text{ , for all j, where } \theta_{ij} = \frac{\pi_i q_{ij}}{\sum_i \pi_i q_{ij}} \qquad (12)$$

In other words, the columns of IP would no longer be centred round 1, making the indicator much more difficult to interpret.

4. Analysis of Price and Quantity Structures

4.1 Before calculating the indicators IP and IQ for the OECD93 data, it is necessary to choose an appropriate GGK index to use as base for the indicators. The particular GGK index which is used here as base is the C(0.5) index. This index has been chosen since, as will be seen, the C(0.5) index occupies a central position among the group of indices studied.

4.2 The values of IP and IQ have accordingly been calculated for the OECD93 data. The 202 expenditure items in the full data set fall into two distinct categories. 199 of the items relate to the consumption of goods and services, and are effectively non-negative. The remaining 3 items, (net purchases abroad by residents: changes in stocks: and net exports), are balancing items, which reflect the differences between total consumption and GDP: these balancing items may be either positive or negative. It is well known that negative expenditures can pose problems for specific aggregation methods, and special techniques may be required to deal with these items. Since this paper is not concerned with the theory of how to handle negative expenditure items in aggregation, this analysis has been restricted to the 199 consumption items only.

4.3 A natural first use of the calculated IP and IQ values is to identify extreme values. Of course, the fact that a value is an outlier does not mean it is necessarily wrong: but extreme values are natural candidates for vetting. To illustrate the kind of extreme values experienced, Tables 1A and 1B show the ten largest, and the ten smallest IP and IQ values, respectively, (ignoring 3 small negative IQ values, and a number of IQ values which are 0).

Table 1A: The Ten Highest and Ten Lowest IP Values

NZL	Dried vegetables	13.12	ICE	Medical analyses	0.32
ICE	Services of nurses	6.04	NOR	Other household services	0.31
CAN	Domestic Services	3.36	ICE	Town gas and natural gas	0.29
TUR	Heaters and air-cond	3.35	UK	Services of nurses	0.28
JAP	Dried vegetables	3.30	TUR	Repair of house appl.	0.28
JAP	Liquefied petroleum gas	3.25	ICE	Liquid fuels for heat.	0.27
TUR	Sporting and recreation	3.16	TUR	Domestic Services	0.27
TUR	Passenger vehicles	3.15	NZL	Other purchased transport	0.25
TUR	Other cereal products	3.15	NZL	Long distance coach/rail	0.22
ICE	Wine (not fortified)	3.12	TUR	Repair of househld. text.	0.21

Table 1B: The Ten Highest and Ten Lowest IQ Values

NOR	Boats, steamers,etc	29.47	SPA	Flowers, plants etc	0.01
TUR	Dried vegetables	18.30	NZL	Spectacle lenses etc	0.01
TUR	Coal, coke etc	14.44	IRE	Other transport routes etc	0.01
SWI	Motorless garden app.	14.39	PRT	Maintenance etc	0.01
GRC	Condensed milk etc	12.66	SPA	Maintenance etc	0.01
AUS	Other meat	12.00	ICE	Services of nurses	0.01
PRT	Wine (not fortified)	11.64	UK	Services of specialists	0.01
TUR	Flour and other cer.	11.41	SPA	Radios and licence	0.01
ICE	Fresh lamb etc	11.07	NOR	Liquefied petroleum gas	0.01
PRT	Dried fish etc	11.05	TUR	Preserved fish & seafood	0.01

4.3 A more consistent approach towards the identification of extreme values would involve looking at the statistical distribution of IP and IQ in each country. It turns out that, for many countries, the distribution of IP and IQ is approximately lognormal: (see, for example, Chart 1, which shows that a lognormal probability plot of the IP values for a typical country, Austria, is approximately linear, with some evidence of over-dispersion at the tails of the distribution: i.e., some excess of very small and very large values.) The fact that the distribution of IP and IQ values tends to be approximated by a standard statistical distribution could make the identification of outliers on a consistent basis easier: though, of course, outliers are not necessarily “wrong”, and, conversely, there may be mistakes in the data which do not appear as outliers.

4.4 Turning now to more structural features of the data, Table 2 gives a useful summary statistic for each country, in terms of the root mean square deviation from 1 of IP and IQ. This is a convenient single summary statistic for measuring by how much the structure of prices, or quantities, in a particular country departs from a simple multiplicative model.

Table 2: Summary Statistics for IP and IQ

Country	**IP root mean squ.dev**	**IQ root mean squ.dev**	**Country**	**IP root mean squ.dev**	**IQ root mean squ.dev**
GER	0.203	1.005	AUT	0.255	0.876
FRA	0.214	0.757	SWI	0.267	1.267
ITA	0.287	0.992	SWE	0.309	0.811
NLD	0.222	1.040	FIN	0.388	0.991
BEL	0.248	0.984	ICE	0.623	1.306
LUX	0.283	1.263	NOR	0.391	2.226
UK	0.229	0.741	TUR	0.747	2.352
IRE	0.314	1.037	AUS	0.404	1.121
DNK	0.269	1.312	NZL[1]	0.358	1.172
GRC	0.453	1.789	JAP	0.554	0.893
SPA	0.332	1.216	CAN	0.340	0.650
PRT	0.599	1.596	USA	0.370	0.481

1. The values for New Zealand have been corrected for the anomalous price for dried vegetables.

4.5 Countries with relatively high values of root mean square deviation from 1 for IP are Turkey (.747), Iceland (.623), Portugal (.599), Japan (.554), and Greece (.453). As regards the indicator IQ, the countries with the highest root mean square deviation from 1 are Turkey (2.35), Norway (2.23) and Greece (1.79). Overall, there is a correlation of 0.57 between the root mean square deviation from 1 of IP and IQ: this is significant at the 0.5% level, and suggests that, if a country's price structure deviates from the simple multiplicative model, then its quantity structure will tend to deviate also.

4.6 The next stage in the analysis was to examine the correlations between countries for the price indicator IP. In fact, since the variable IP is not symmetric about 1, (with deviations below 1 restricted to the range from 0 to 1, while deviations above are unbounded), the correlations were calculated between the values of log(IP), rather than the raw values . To avoid possible distortions posed by extreme values of IP, values of IP below .333 or above 3 were excluded from the analysis.

4.7 The full correlation matrix is not reproduced here for reasons of space, but may be accessed, (along with other detailed tables) at the website http://www.blackwellpublishers.co.uk/rss/ , or in Cuthbert, [2000]. Most of the correlations are positive: none of the correlations is particularly large, with the largest being Finland/Norway at 0.55. Given that the test statistic for the hypothesis of zero correlation is approximately 0.14, (at the 5% significance level), many of the pairwise correlations would be significantly different from zero if tested separately.

4.8 To get a clearer impression of the correlation structure, the correlation matrix has been summarised, by drawing a dendrogram: see Chart 2. The dendrogram has been constructed on a nearest neighbour basis: the simplest way to interpret it is as follows. The scale of the vertical axis represents correlations: for any two countries, find the lowest point on the path on the dendogram joining the two countries: then it is possible to find a chain of countries which joins the two countries in question, and where every correlation between neighbouring countries along the chain is larger than the given lowest point.

4.9 Examination of the dendrogram suggests a number of interesting features about the correlation matrix. First of all, geographically neighbouring countries tend to have relatively similar price structures. Secondly, there are, in addition, suggestions of distinct groups of countries: in particular, there is a Mediterranean group of Portugal, Turkey, Spain, Greece and Italy: and there is a Scandinavian group of Finland, Norway, Sweden, Denmark and Iceland. There is also a weaker group embracing Europe and Australasia. Japan has a price structure which is relatively distinct from other countries.

4.10 The next stage was to carry out a similar analysis of the correlations between countries for the quantity indicator, IQ. Again, correlations were calculated using the logarithms of the IQ data: and, again, very small and very large values of IQ were excluded from the analysis. In this case, since the spread of IQ tends to be larger, a lower cut-off of 0.1 was taken, and an upper cut-off of 6. Again, most correlations are positive, with none particularly large. The dendrogram for the correlation matrix is given in Chart 3. Interestingly, the broad picture which emerges from Chart 3 is similar to that already observed for the price based correlations, though there are differences in detail. Geographically neighbouring countries tend to have similar quantity structures: there is again a distinct Mediterranean group, though this group is now somewhat less cohesive: the Scandinavian group still exists, but is again somewhat less clear cut, and merges in to the rest of Europe. Canada is detached from the US, and has now joined a small sub-group with Australia, New Zealand and Ireland. The US and Japan have quantity structures which are relatively dissimilar from each other, and from all other countries.

4.11 The final topic in this section is an examination of whether there is evidence, from the IP and IQ data, of negative correlation between prices and quantities within individual countries. Accordingly, the correlations between log(IQ) and log(IP) values were calculated, for all possible pairs of countries, (using the same upper and lower cut-offs as employed earlier for the log(IP) and log(IQ) correlations). The resulting correlation matrix shows clear evidence of negative correlation between the price and quantity structures in each country. The relevant terms in the correlation matrix are on the diagonal: all of these diagonal terms are negative, ranging from -0.15 (Sweden) to -.049 (Japan). Moreover, in each row of the table, the diagonal element is the largest negative element: in other words, for each country, the quantity indicator for that country is more negatively correlated with the price structure of that country, than with the price structure of any other country.

5. Relating the Data Structure to the Results of Different Aggregation Methods

5.1 The next stage is to examine the comparative results of applying a number of different aggregation methods to the data in order to compute direct volume estimates for each country, and then to consider how the observed features of the data structure relate to the comparative behaviour of the different aggregation methods. The aggregation methods considered were the GEKS, the GK, the Iklé , the equal weighted Van Ijzeren, and the C(α) indices for α = 0.1,0.2,… 0.9. In each case the US was taken as the numeraire country. The detailed results are not repeated here, for reason of space, but can be accessed in Cuthbert, [2000], or at the website referred to in para 4.7.

5.2 A useful way of exploring the inherent structure in these detailed results is to consider the volume estimates set out in an array, with countries as columns, and the different aggregation methods as rows. The correlation can then be calculated between the different columns in this array: that is the correlations can be calculated between the volume estimates for each pair of countries, (excluding the US, since, as base country, its volume estimates are constant by construction.) All of the resulting correlations are positive, and for many country pairs, the correlation between the different volume measures is extremely high, typically larger than 0.9 - indicating that the different volume measures are moving consistently for these countries. However, correlations involving certain countries tend to be lower- this is the case, in particular, for Turkey, Portugal, UK and Japan.

5.3 An analysis of the principal components of the correlation matrix shows that it has a simple structure. If the scaled volume measures are regarded as points in country space, then the first principal component is that linear combination of countries which accounts for most of the dispersion in the data: the second component is that direction, at right angles, which accounts for most of the remaining dispersion, and so on. The first two principal components account for almost all, (over 99%), of the variability in the data. The first principal component alone accounts for 91% of the variability in the data. This principal component is an almost equally weighted sum of the different countries. The second principal component has large positive weights for Turkey, Japan and Portugal, and a particularly large negative weight for the UK, (and, to a lesser extent, for Austria, Sweden, Germany and Italy). The values of the first and second principal components can be calculated for each index, and it is instructive to plot these as in Chart 4, giving a visual representation, in two dimensions, of the structure of the country volumes. The chart shows how the first principal component moves from high values for the GK, through the C(α) indices, to low values for the Iklé and Van Ijzeren indices: in other words, the first principal component, (which accounts for most of the correlation structure), is dominated by a consistent pattern, with the GK for all countries, (apart from the US), giving higher volume estimates than the C(α) indices with higher α values. The second principal component is in many ways more interesting. It is dominated by the large negative value for the GEKS, compared with relatively small positive values for the other indices. Recalling the countries which dominate the component scores for the second principal component, the second principal component reflects the extent to which the GEKS is relatively higher than the other indices in certain countries, (in particular, the UK, but to a lesser extent Austria, Sweden etc.): and conversely, the GEKS is relatively lower than the other indices in another group of countries, (particularly Turkey, but also Japan and Portugal).

5.4 The evidence discussed in para 4.11 of negative correlation between the price and quantity structures in each country provides a plausible heuristic explanation of the form of the first principal component. This principal component reflected the way in which the volume measures of all countries apart from the US were relatively larger for the GK index, and low index $C(\alpha)$ indices, as compared with the Iklé and high index $C(\alpha)$ indices. The GK and low index $C(\alpha)$ indices give greater weight to large countries in calculating international prices. Given the US is by some margin the largest country, this means that international prices for the GK and low index $C(\alpha)$ indices will tend to be relatively closer to the US structure of prices. Given negative correlation between US prices and quantities, this means that GK international prices will tend to value the US quantities relatively lowly, (or putting this another way, since US volumes are normalised, will tend to value other countries quantity structures relatively highly), so accounting for the observed form of the first principal component of the volume correlation matrix. In other words, the first principal component relates to a classic Gerschenkron type effect.

5.5 We now consider the question of whether there are any factors in the basic data which explain the form of the second principal component of the country volume matrix, which, it will be recalled, reflects differences between countries in the relativity between the GEKS and GGK indices. There are indeed features of the underlying price and quantity structures which account for this behaviour.

5.6 The key to the relationship between the GEKS and GGK indices lies in the following identity, namely

$$GEKS_{jk} = g_j \, g_k^{-1} \, . \, GGK_{jk} \qquad (13)$$

The derivation of this identity, and the definition of the positive terms g_j, are given in the Annex.

5.7 It is possible to develop a heuristic argument, (not repeated here, for reasons of space), which suggests that the magnitude of the terms g_j is likely to be related in a simple way to the behaviour of the price and quantity structure indicators IP and IQ, given the observed negative correlation between IP and IQ. This argument suggests that, if a particular country j has price and quantity structures close to 1, (in the sense that the root mean square deviations from 1 of IP and IQ are small), then the term g_j will tend to be relatively large. Conversely, if IP and IQ deviate markedly from 1, then g_j is likely to be small. This heuristic argument is indeed borne out by examining the relationship between g_j and the root mean square deviations from 1 of IP and IQ in Table 2. In fact, the simple correlation between g_j and the root mean square deviation from 1 of IP_{ij} is -0.85: the simple correlation between g_j and the root mean square deviation from 1 of IQ_{ij} is -0.73: regression of g_j on the root mean square deviations of IP_{ij} and IQ_{ij} explains 80% of the variation in g_j, with the coefficients of both variables in the regression being significantly different from zero.

5.8 The conclusion to be drawn from the above is that the structural feature in the price and quantity data which accounts for most of the relative differential between the GEKS and GGK indices relates to how far the price and quantity indicators in a given country depart from unity. Countries with large departures, like Turkey, Portugal and Japan, will tend to do relatively worse under the GEKS than the relevant GGK index, whereas countries with small departures, like the UK, will be in the opposite position. Note that a large root mean square deviation for IP or IQ does not represent an intrinsic property of a country taken in isolation: but reflects how different the country's prices or quantities are from international average prices, or average international quantities. It is a truism, but nevertheless worth stating, that countries with large root mean square deviations will tend to be those which, for geographical or other reasons, tend to be furthest from the "average".

6. Robustness of Price and Quantity structure Indicators

6.1 For most of this paper, the indicators IP and IQ have been calculated relative to the C(0.5) base. The C(0.5) index is a natural first choice as a base index, given its central position among the C(a) indices, as is illustrated, for example, by Chart 4. This section looks at robustness under different choices of base index. To test the robustness of the IP and IQ indicators relative to different choices of base, the analyses described in Section 4 above were repeated, first of all for indicators calculated relative to the GK as base, and secondly, relative to the Iklé. Fuller results of these variant analyses are described in Cuthbert, [2000]. The conclusion which emerges from this work is that the major price and quantity features revealed by the use of the IP and IQ indicators are not sensitive to the particular GGK index chosen as base for the indicators. Such features as do vary with choice of base are fairly limited, and vary predictably.

7. Conclusion

7.1 In conclusion, indicators of price and quantity structure, as defined here, provide a tool which is capable of revealing structural features of the basic data: such features include natural groupings of countries on the basis of similar price and quantity structures, and evidence of negative correlation between prices and quantities. Such features are not only of potential interest and importance in themselves, but also explain what underlies the observed comparative performance of different aggregation methods.

References

Cuthbert, J. R. (1999) Categorisation of additive purchasing power parities. *Review of Income and Wealth*, 45, 235-249.

Cuthbert, J. R. (2000) Theoretical and practical issues in purchasing power parities illustrated with reference to the 1993 Organisation of Economic Co-operation and Development Data. *Journ. Roy. Stat. Soc., Series A*, 163 , 421-444.

Geary, J. (1958) A note on the comparison of exchange rates and purchasing power between countries. *Journ. Roy. Stat. Soc.*, 121, 97-99.

Iklé, D. M. (1972) A new approach to index number theory. *Quarterly Journal of Economics*, 86, 188-211.

Van Izjeren, J. (1987) *Bias in international index numbers: a mathematical elucidation.* Eindhoven: CIP-Gegevens Koninklijke Bibliotheek.

Annex: The derivation of the Identity in Equation (13)

A typical building block in defining the GEKS index is of the form $\sum_i p_{ij} q_{il}$, say. This can be rewritten as follows:

$$\sum_i p_{ij} q_{il} \;=\; e_j^{-1} \sum_i \{ \frac{e_j p_{ij}}{\pi_i} \frac{\pi_i q_{il}}{\sum_i \pi_i q_{il}} \} \;\sum_i \pi_i q_{il}$$

$$= \; e_j^{-1} \sum_i IP_{ij} \theta_{il} \sum_i \pi_i q_{il} \;,$$ (where θ is as defined in equation (11) above).

Substituting this expression into the definition of the Fisher volume index F_{jl}, and recalling that $\sum_i IP_{ik} \theta_{ik} \;=\; 1$, from equation (11), it follows that

$$F_{jl} \;=\; \left[\frac{\sum_i IP_{il} \theta_{ij}}{\sum_i IP_{ij} \theta_{il}} \right]^{.5} GGK_{jl}$$

$$= \left[\frac{\lambda(l,j)}{\lambda(j,l)} \right]^{.5} GGK_{jl} \qquad \text{, say,}$$

where $\lambda(l,j)$ is defined as $\sum_i IP_{il} \theta_{ij}$.

Hence it follows that

$$GEKS_{jk} \;=\; \left[\prod_l \frac{\lambda(l,j)}{\lambda(j,l)} \right]^{\frac{1}{2J}} \left[\prod_l \frac{\lambda(l,k)}{\lambda(k,l)} \right]^{\frac{-1}{2J}} GGK_{jk}$$

$$= \; g_j \; g_k^{-1} \,.\, GGK_{jk}$$

where $g_j \;=\; \left[\prod_l \frac{\lambda(l,j)}{\lambda(j,l)} \right]^{\frac{1}{2J}}$. This is the desired identity.